

7.5 Roots and Zeros

Fundamental Theorem of Algebra

- all polynomial equations with a degree greater than zero have at least one root in the set of complex numbers
- a polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients, has exactly n roots

How many roots?

ex:

$$a - 10 = 0 \quad 1$$

$$x^2 + 2x - 48 = 0 \quad 2$$

$$3a^3 + 18a = 0 \quad 3$$

$$y^5 - y = 0 \quad 5$$

Solve # 3.

$$3a(a^2 + 6) = 0$$

$$a = 0 \quad a^2 = -6$$

$$a = \pm i\sqrt{6}$$

1 Real
2 Imaginary

Solve # 4.

$$y(y^4 - 1) = 0$$

$$y(y^2 - 1)(y^2 + 1) = 0$$

$$y = 0 \quad y^2 = 1 \quad y^2 = -1$$

$$y = \pm 1 \quad y = \pm i$$

3 Real
2 Imaginary

Complex Conjugates

 $a, b \in \mathbb{R} \quad b \neq 0$ If $a + bi$ is a zero, then $a - bi$ is a zero1. If $3 + 2i$ is a zero, then $3 - 2i$ is also a zero.2. If $5 - 3i$ is a zero, then $5 + 3i$ is also a zero.3. If $\sqrt{5}$ is a zero, then $1 + \sqrt{5}$ is also a zero.
(It also works for radicals)4. If $6i$ is a zero, then $-6i$ is also a zero.
 $0 + 6i$

Write a function with the following zeros.

3, $2 + i$, $2 - i$

$$(x - 3)(x^2 - 4x + 5) = 0$$

Write a function with the following zeros.

 $-2, 2 - \sqrt{3}, 2 + \sqrt{3}$

$$2 - \sqrt{3} + 2 + \sqrt{3} = 4$$

$$(2 - \sqrt{3})(2 + \sqrt{3}) = 1$$

$$4 - 3$$

$$(x + 2)(x^2 - 4x + 1) = 0$$

$$x^3 - 4x^2 + x$$

$$2x^2 - 8x + 2$$

$$x^3 - 2x^2 - 7x + 2 = 0$$

Write a function with the following zeros.

-8, $5i$

HW

p375-376

13, 15, 17,

35, 38, 39, 41